

Voltage tuning of the resonance frequency of electroactive polymer membranes over a range of 75%

Philippe Dubois, Samuel Rosset, Muhamed Niklaus, and Herbert Shea*

Ecole Polytechnique Fédérale de Lausanne (EPFL), Microsystems for Space Technologies
Laboratory (LMTS), Neuchâtel, Switzerland

ABSTRACT

We report on a novel technique to control the resonance frequency of polymer membranes, without additional external actuators. An electrostatic force is used to apply compressive stress to a dielectric electroactive polymers membrane, consisting of a 25 micron thick, 1 to 4 mm diameter, polydimethylsiloxane (PDMS) film bonded onto patterned silicon or Pyrex wafers. Both sides of the membranes are rendered conductive by low-energy metal ion implantation. Ion implantation is chosen because it stiffens the membrane much less than sputtering a film of similar thickness [1][2]. The initial resonance frequency of the membrane is given by its geometry, the Young's modulus and stress of the composite film. The technique presented here allows tuning the resonance frequency from this initial value down to zero (at the buckling threshold) by adding compressive stress due to a voltage difference applied to the electrodes on both sides of the membrane. We have measured a reduction of the first mode resonance frequency of up to 77% (limited by dielectric breakdown) for ion-implanted membranes [3]. The tuning is repeatable and allows for continuous variation. Excellent agreement was found between our measurements and an analytical model we developed based on the Rayleigh-Ritz theory.

Keywords: resonance frequency tuning, tunable resonators, ion implantation, dielectric electroactive polymer, EAP, DEAP, DEA, dielectric elastomer actuator, artificial muscle.

1. INTRODUCTION

A common requirement in MEMS-based resonators is precise tuning of the frequency, ideally over a large range. This need has led to a number of developments [4]-[8] for the active tuning of the resonance frequency of silicon beams and plates, mostly by using comb-drives to apply compressive or tensile stress on silicon beams, or by modifying the effective spring constant of the resonator using an additional electrostatic force. These techniques are highly effective and allow a large tuning range, but require large external actuators, generally much larger than the resonator.

In this paper, we report on dielectric membranes whose fundamental resonance frequency (of order 1 to 5 kHz) is continuously reduced by changing its stress by means of an electrostatic pressure applied by electrodes on both sides of the membrane. The mechanical compliance of a membrane being directly linked to the resonance frequency, such a device could for example be used to tune the attenuation of an acoustic wave propagating in air.

The resonating membrane discussed here consists of a dielectric electroactive polymer, which is an elastomer sandwiched between two compliant electrodes. The tuning principle is to apply a voltage across the membrane, thus adding compressive stress due to the electrostatic force, hence effectively softening the device and reducing the resonance frequency. The tuning can in principle reduce the resonance frequency to zero at the buckling threshold.

Several authors have reported actuators based on DEAPs, for example [9]-[16], the resonance modes of actuated pre-stressed DEAPs membrane have not yet been reported except by our group in [3]. We have used polydimethylsiloxane (PDMS) as the elastomer, which, combined with standard micromachining processing, allows short response times, and resonance frequencies in the audible range (400 Hz to 5 kHz). We have developed a low-energy ion implantation technique that allows patterning compliant electrodes in the surface of elastomer membranes [17][1]. This technique allows for high electrode compliance and for strains of over 175% to be applied while remaining conductive [2], and hence allows achieving displacements of over 25% for mm-scale EAP devices [18]. The operating principle of a

* herbert.shea@epfl.ch ; <http://lmts.epfl.ch>

buckling mode actuator is shown in Figure 1 and in Figure 2. The regime discussed in this paper is limited to the pre-buckling region.

PDMS is a very soft elastomer, with a Young's modulus of order 1 MPa. The devices reported here have a resonant frequency that could be reduced by more than 40% from 2.46 kHz and by 77% from 1.6 kHz. The same technology (compliant metal electrodes implanted on both sides of a polymer membrane or beam) would also allow the fine tuning of the resonance frequency of much stiffer polymers.

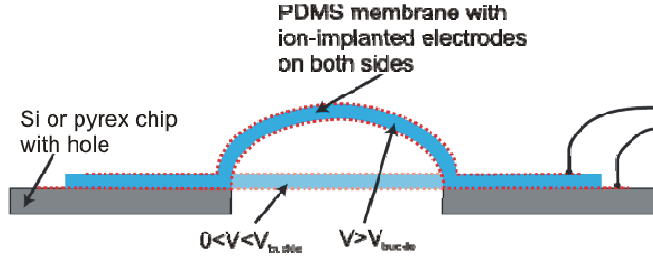


Figure 1: Schematic diagram of the operating principle of a buckling mode EAP actuator. Frequency tuning is possible in the voltage regime below buckling (see Figure 2).

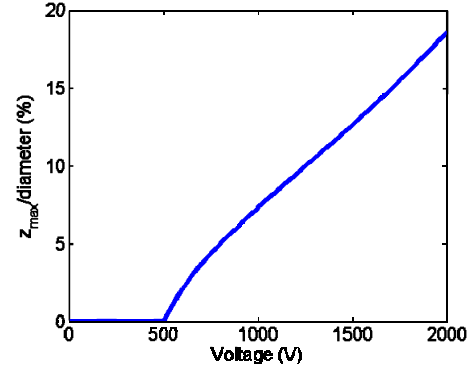


Figure 2: Theoretical plot of vertical displacement of the center of a membrane vs. applied voltage, showing a buckling threshold at 500 V for this example.

2. ANALYTICAL MODEL

The basic concept is that the resonance frequency f_{res} of membranes can be reduced by adding compressive stress. For our EAP case, the initial tensile stress due to fabrication can first be reduced to zero, then made compressive by adding an electrostatic stress using a voltage applied to compliant electrodes on both sides of the membrane (see Eq. 8).

For a membrane, there are two main limit cases for which the resonance frequency can be rigorously derived: bending (no stress), and drum mode (stress dominated). Timoshenko *et al.* derived the resonance frequency of plates in the bending case (zero stress, the flexural rigidity dictates f_{res}) [19], and of tensile stressed membranes or drums (only stress contribution, no bending dependence, the stress dictates f_{res}). We extended their formalism to the more realistic intermediate case of membranes of finite thickness, using the Rayleigh-Ritz approximation to combine the contribution from the bending and stretching energies. A full derivation is given in [3], and is summarized below.

The Rayleigh-Ritz method allows accurate estimations of the resonance frequency of compliant systems. With the key approximation of equal mode shapes for plate and drum, the total pulsation can be written as:

$$\omega_{res}^2 = \omega_{plate}^2 + \omega_{drum}^2 \quad \text{Eq. 1}$$

where the plate pulsation ω_{plate} can be expressed as a function of the membrane diameter d , thickness h , membrane homogenous density ρ and flexural rigidity D :

$$\omega_{plate}^2 = 2 \frac{U_{plate,max}}{\int_{volume} \rho(x,y) \cdot [z(x,y)]^2 dVol} = \left(\frac{40.704}{d^2} \right)^2 \frac{D}{\rho h} \quad \text{Eq. 2 [19],[20]}$$

With the flexural rigidity D that depends on the Young's modulus Y and Poisson ratio ν given by:

$$D = \frac{Yh^3}{12(1-\nu^2)}, \quad \text{Eq. 3}$$

The drum pulsation ω_{drum} as a function of the stress $\sigma_{x,y}$, and diameter d is given by:

$$\omega_{drum}^2 = 2 \frac{U_{drum,max}}{\int_{volume} \rho(x,y) [z(x,y)]^2 dVol} = \left(\frac{4.81}{d} \right)^2 \frac{\sigma_{x,y}}{\rho} \quad \text{Eq. 4 [19],[20]}$$

So we have the resonance frequency f of a stressed membrane that is not perfectly thin, i.e., a plate, using the Rayleigh-Ritz method:

$$f = \sqrt{f_{plate}^2 + f_{drum}^2}$$

$$f = \frac{1}{2\pi d \sqrt{\rho}} \sqrt{23.13 \sigma_{x,y} + \frac{1657 D}{d^2} \frac{D}{h}} \quad \text{Eq. 5}$$

The initial stress state in a bi-axially uniformly tensile stressed membrane is given by $\sigma_{0x} = \sigma_{0y} = \sigma_0 > 0$ and $\sigma_{0z} \cong 0$

Assuming there is no displacement of the membrane (i.e., no buckling), the stress produced by the electrostatic force is hydrostatic, i.e. there are only identical terms in the diagonal of the stress tensor $[\sigma_E]$ [25]. The total stress is the sum of the initial stress $[\sigma_0]$ and the electrostatic stress $[\sigma_E]$:

$$[\sigma] = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \sigma_E & 0 & 0 \\ 0 & \sigma_E & 0 \\ 0 & 0 & \sigma_E \end{bmatrix} = \begin{bmatrix} \sigma_0 + \sigma_E & 0 & 0 \\ 0 & \sigma_0 + \sigma_E & 0 \\ 0 & 0 & \sigma_E \end{bmatrix} \quad \text{Eq. 6}$$

The electrostatic stress produced by the actuation voltage V given by (Eq. 7):

$$\sigma_z = \sigma_E = -\varepsilon \frac{V^2}{h^2} \quad \text{Eq. 7 [12]}$$

where ε is the permittivity of the elastomer. Therefore, according to

Eq. 6, the stress in the plane x,y when the membrane is actuated becomes (Eq. 8):

$$\sigma_x = \sigma_y = \sigma_0 - \varepsilon \frac{V^2}{h^2} \quad \text{Eq. 8}$$

Combining Eq. 5 with Eq. 8, we obtain the first mode resonance frequency of an actuated circular membrane:

$$f = \frac{1}{2\pi d \sqrt{\rho}} \sqrt{23.13 \left(\sigma_0 - \frac{\varepsilon}{h^2} V^2 \right) + \frac{1657 D}{d^2} \frac{D}{h}} \quad \text{Eq. 9}$$

From Eq 9, one sees that the resonance frequency can be tuned down from its initial value by applying a voltage on the electrodes. According to this model, the membrane resonance frequency can be voltage tuned from f_{high} to f_{low} :

$$f_{no_stress} \leq f_{actuated} \leq f_{high}, \text{ where}$$

$$f_{high} = \frac{1}{2\pi d \sqrt{\rho}} \sqrt{23.13 \sigma_0 + \frac{138.1}{d^2} \frac{Y h^2}{1 - \nu^2}}$$

$$f_{no_stress} = \frac{1}{2\pi d \sqrt{\rho}} \sqrt{\frac{138.1}{d^2} \frac{Y h^2}{1 - \nu^2}}$$

We thus have an expression (Eq. 9) for the resonance frequency vs. applied voltage, depending on parameters of the membranes that can all be measured or determined. This is plotted for three examples in Figure 3.

The higher the initial stress is, the higher the frequency span becomes. But in practice resonance frequencies below f_{no_stress} corresponding to compressive stresses below the buckling threshold can also be reached, all the way own to zero at buckling. On a real membrane the limitation comes generally from the electrical breakdown, thus preventing tuning the frequency to zero.

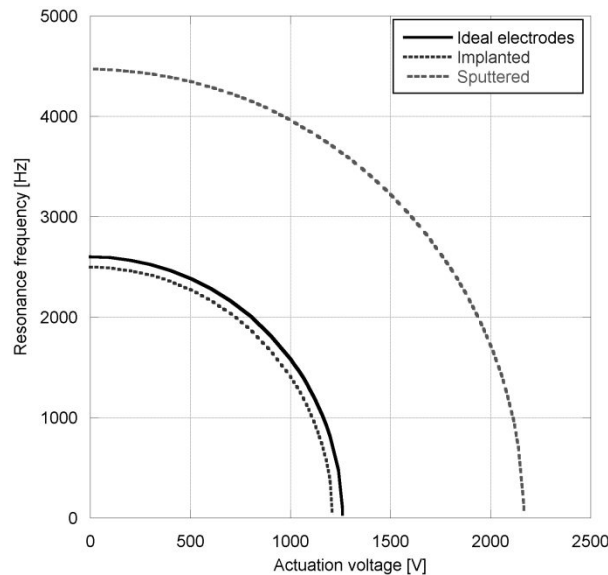


Figure 3: Calculated resonance frequency vs. applied voltage to illustrate the influence of the stiffening of the membrane due to the electrodes from Equation 9. The plots are generated using typical parameters measured on a 2-mm-diameter 28.6 μm thick membrane. Bare PDMS ($Y=1$ MPa, $\sigma_0=50$ kPa) (corresponds to ideal electrode curve where electrodes do not induce any stiffening), Gold ion implanted PDMS ($Y=2.6$ MPa, $\sigma_0=43.4$ kPa), Gold sputtered PDMS ($Y=7$ MPa, $\sigma_0=141$ kPa).

3. FABRICATION & DESIGN

3.1 Device Fabrication

The dielectric electroactive actuators (DEA) studied here consist of a dielectric EAP (DEAP) circular membrane fixed to a rigid Pyrex or silicon frame (schematic cross-section in Figure 1 and photo of actual devices in Figure 4). The membrane is a PDMS film with nm-thick gold electrodes on both sides. The fabrication process of the polydimethylsiloxane (PDMS) membrane induces a tensile stress that, along with membrane geometry, membrane density and Young's modulus, determines the initial resonance frequency.

Circular holes, 2 to 4 mm in diameter, were patterned by powder blasting in Pyrex and by DRIE in 4" silicon wafers. The PDMS layer (Dow Corning Sylgard 186) was spun on a transfer film stretched on a rigid frame. Once the PDMS polymerized, the PDMS membrane was bonded on a patterned Si or Pyrex chip after plasma O_2 activation and removed from the transfer film [1] [19].

For a high performance EAP device, it is necessary to have compliant electrodes: a) that can repeatedly sustain strains of over 30%, and b) whose stiffness is smaller than that of the elastomer. This last point is not trivial given that metals have a Young's modulus 4 orders of magnitude larger than the elastomers we use. Ion implantation of gold can rapidly produce low resistivity compliant electrodes that can be patterned on the micron scale on polymers [2]. Other techniques that have been used with success on the sub-100 μm scale are stamping of carbon powder [21], and patterned deposition of thin metal layers [22]. Ion implanted electrodes offer long lifetime, easy patterning using photoresist or a steel shadow-mask, and EAP devices using such electrodes have been shown to operate unchanged after more than 4 million

cycles. Ion implantation leads to roughly an additional 1 to 2 MPa stiffness of a 30 μm thick Sylgard 186 PDMS film [23].

Details of the implantation process and parameters are given in [2]. For the devices reported here, gold ions with energy of 5 keV and at doses of $2 \cdot 10^{16}$ ions/ cm^2 were used for both the top and bottom electrodes. Sputtered gold layers 8 to 15 nm thick were also tested to compare the impact of this method versus implantation on the resonance frequency of the PDMS membranes.



Figure 4: Picture of a microfabricated electroactive tunable resonance frequency membranes. Left, 2 mm diameter membrane on a Pyrex chip with top and bottom gold contracts to the implanted electrodes. Right, 3 mm diameter membrane on a silicon chip making use of the silicon as the electrical contact to the lower implanted electrodes. For both devices the compliant electrodes on the membrane were fabricated using gold ions at 5 keV with a dose of 2×10^{16} as determined by Rutherford back-scattering.

3.2 Influence of electrode type on resonance frequency

Figure 3 illustrates, based on the analytical model (Eq. 9), the effect of electrode compliance on the frequency span for three different electrode types: ideally compliant, ion implanted and sputtered electrodes. The Young's modulus Y and residual stress σ_0 were measured using a custom-built bulge test apparatus for soft elastomers [24],[27],[28] on 2 mm diameter membranes. The values are for the electrode-elastomer-electrode composite. The parameters of the tested samples are summarized in Table 1.

	Implanted Au electrodes	Sputtered Au electrodes
Diameter d [mm]	2	2
Measured thickness h [μm]	28.6	20.4
Density ρ [kg/m^3]	1120	1120
Relative dielectric permittivity ε	3	3
Measured average Young's modulus Y [MPa]	2.6	7.54
Measured average initial stress σ_0 [kPa]	43.4	140.94
Calculated initial resonant frequency in mode 1 f_{high} [Hz]	2496	4387

Table 1: Parameters extracted from the devices and then used in Eq 9. Young's modulus and initial stress are measured by bulge test, membrane thickness is measured with an optical profilometer. The Dow Corning Sylgard 186 PDMS properties are given by the manufacturer.

Sputtered gold electrodes significantly stiffen the membrane, increasing the overall Young's modulus by a factor of 5 to 20 depending on deposition conditions, and increasing the residual stress by a factor 2 to 3. This increases the initial resonance frequency, and hence also the voltage required for full frequency tuning. The thickness of very thin sputtered layers (<20 nm to be as thin as possible while being conductive), required by this application, is difficult to control and

has a major impact on the overall membrane properties. Based on Eq. 9, the influence of the stress σ_0 and Young's modulus Y can be estimated from the model (Eq. 9, see Figure 3).

For ion implanted gold electrodes, we observed an increase of the overall Young's modulus due to ion implantation of about a factor 2-3 and a small modification of the inner overall stress of $\pm 20\%$. This small modification of the stress allows designing an actuator with an accurately pre-defined initial resonance frequency.

4. RESONANCE FREQUENCY MEASUREMENTS

The resonance frequency of PDMS membranes was measured by applying an acoustic wave with a loudspeaker on one side of the membrane and measuring the membrane displacement with a laser Doppler vibrometer (Polytec MSV400) as the frequency was slowly swept. These measurements were done with an actuation voltage ranging from 0 to 1.8 kV. Electrical fields larger than 50 V/ μm are larger than the typical breakdown field for commercially available PDMS, but are typical of thin elastomer membranes [23].

A typical resonance quality factor Q of 45 was measured in air on a 3-mm-diameter membrane. Q decreases as the resonance frequency is tuned down by increasing the voltage. The decrease was by a factor of two for 75% reduction in resonance frequency.

The initial resonance frequency for two membranes was as predicted by Eq. 9 (with $V=0$) within 1.5% (note there are no free parameters). For sputtered and ion implanted electrodes we observed the expected shift of the resonance frequency produced by electrostatic actuation. For the Au ion implanted 2-mm-diameter sample actuated up to 1000 V (Figure 5), the relative resonance frequency shift was -43% (from 2463 Hz down to 1406 Hz), and for the sputtered membrane -20% (from 4450 Hz down to 3563 Hz) at 1100 V on a same size sample (Figure 6). Maximum voltages were limited by the need to stay below the dielectric breakdown field. For a third ion-implanted sample with a 4 mm diameter membrane, we reached a 77% reduction in resonance frequency, from 1620 Hz to 530 Hz at 1.8 kV [3]. The spread between samples is due to variations in Young's modulus and stress due to different curing conditions for the PDMS.

The solid line in Figure 5 and Figure 6 correspond to equation 9, using the average parameters (Y , σ_0 , h) measured by bulge test and profilometer after electrode fabrication. An excellent match ($<1.5\%$ difference) between model and measurements on ion implanted samples was observed, despite having no fitting parameters. This excellent agreement between the model and the measurements was verified on many samples having membrane thicknesses ranging from 20 to 35 μm and diameters varied between 2 and 5 mm.

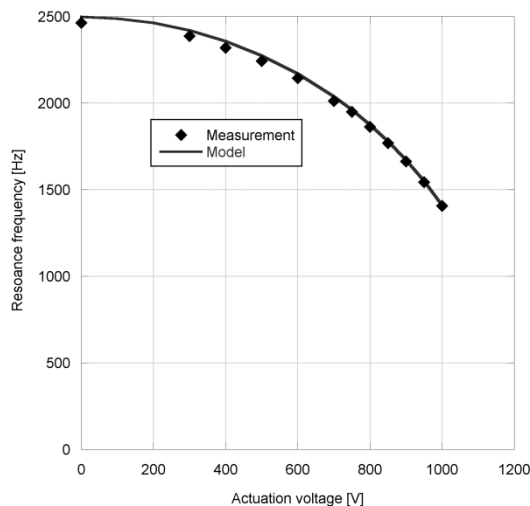


Figure 5: Resonance frequency as a function of actuation voltage for a 2-mm diameter membrane with Au ion implanted electrodes. The diamonds correspond to measurements, and the solid line is computed from the model of equation 9.

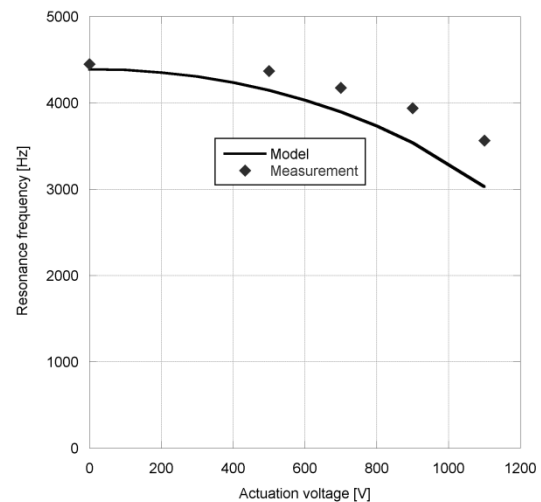


Figure 6: Resonance frequency as a function of actuation voltage for a 2-mm diameter membranes with Au sputtered electrodes. The diamonds correspond to measurements, and the solid line is computed from the model of equation 9.

On sputtered samples (Figure 6) a larger discrepancy between model and data is observed at high actuation voltages, up to 15% at high voltages. The sputtering process leads to a composite film with properties significantly different from the virgin PDMS, and could introduce effects the model does not take into account, such as stress and Young's modulus gradients.

5. CONCLUSIONS

The resonance frequency of microfabricated dielectric EAP PDMS membranes was repeatedly and reversibly voltage controlled up to a 77% range. Two types of compliant electrodes were tested on the PDMS membranes: very thin (10 to 20 nm) sputtered gold, and implanted gold (5 keV energy from an FCA implanter with doses of order $2 \cdot 10^{16}$ atoms/cm²). For the ion implanted devices, an actuation voltage of 1.8 kV enabled a change in the resonance frequency of the membrane of more than 75%. Sputtered electrodes have a smaller achievable tuning range because of the stiffening that requires a higher actuation voltage, which is larger than the PDMS breakdown voltage.

An analytical model that was developed for the voltage dependence of the resonance frequency, describing the entire range of actuation going from 0 V to the buckling voltage. This model is more accurate in the tensile domain than the existing model for thick plates [26] applied to DEAPs and gave excellent agreement with the measured data for ion-implanted samples. The slightly lower accuracy of the model observed for less compliant sputtered electrodes reveals the approximation of the model that does not consider gradients of stress and Young's modulus.

The technique reported here for PDMS can be extended to any dielectric elastomer, such as the acrylic VHB commonly used in EAP actuators, and also to other polymers commonly used in microfabrication such as SU8 or PMMA. By actively varying their resonance frequency (and hence their compliance) the membranes can be used as frequency-tunable attenuators with possible applications in active filters, analog sound processing and active damping. The same technology could also allow the fine-tuning of the resonance frequencies in the MHz range of devices made from much stiffer polymers.

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